A GENERALIZED ALGORITHM FOR THE MODELING OF THE DISPERSIVE CHARACTERISTICS OF MICROSTRIP, INVERTED MICROSTRIP, STRIPLINE, SLOTLINE, FINLINE, AND COPLANAR WAVEGUIDE CIRCUITS ON ANISOTROPIC SUBSTRATES.*

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ABSTRACT

Substrate materials typically used for microwave or millimeter wave applications may exhibit inherent anisotropy or may be chosen to be anisotrophic for some particular device. This paper presents techniques which compute the dispersive properties of slotlines, coplanar waveguides, finlines, inverted microstrip and striplines on anisotrophic substrates.

I. INTRODUCTION

Anisotropy is inherent to many substrate materials typically used for microwave or millimeter wave applications. Since it influences the circuit parameters of interest, it should be included in the modeling of the substrate effects on the circuit under investigation. Furthermore, anisotropy may prove beneficial for certain applications (e.g. equilization of even-odd mode phase velocities) and therefore should be investigated rigorously. The case of microstrip on anisotropic substrates has been investigated to a certain extent (1)-(4), including some results for microstrip discontinuities (3). In addition, a quasistatic solution has recently been reported for coplanar waveguides on anisotropic substrates (5).

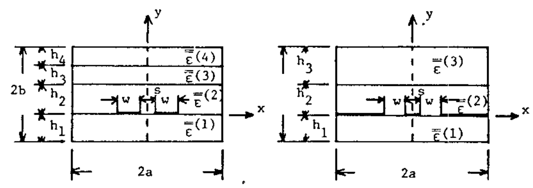
This article addresses the problem in a more generalized fashion. In particular, an approach is presented (4) which has been modified to characterize the dispersive properties of microstrip, inverted microstrip, striplines, slotlines, finlines and coplanar waveguides for single as well as coupled transmission line systems. Examples are given for sapphire, epsilam and boron nitride anisotropic materials. Multiple layers, such as an overlay on substrate, are also included in this generalized algorithm.

II. METHOD OF SOLUTION

It is assumed that the relative permittivity of the i^{th} layer is given by

$$\bar{\bar{\varepsilon}}^{(i)} = \begin{pmatrix} \varepsilon_t^{(i)} & 0 & 0 \\ 0 & \varepsilon_y^{(i)} & 0 \\ 0 & 0 & \varepsilon_t^{(i)} \end{pmatrix} , \tag{1}$$

where $\varepsilon_t^{(i)}$ is the relative permittivity on the xz-plane. A typical geometry is shown in Figure 1 for coupled microstrip and coplanar waveguide.



(a) Coupled Microstrip (b) Coplanar Waveguide

Figure 1.

Coupled Microstrip, Coplanar Waveguide Geometries on Anisotropic Layers

The current distribution (for coupled microstrip) or the field distribution (in the case of coplanar waveguide) will be determined by employing an even-odd mode representation of the electromagnetic field in terms of Fourier Transforms in conjunction with the method of moments.

For microstrip even modes or coplanar waveguide odd modes the fields are:

$$\tilde{E}_y^{TM}(k_n) = \int_0^a E_y^{TM}(x)\cos(k_n x)dx , \tag{2}$$

and

$$\tilde{H}_y^{TE}(k_n) = \int_0^a H_y^{TE}(x)\sin(k_n x)dx . \tag{3}$$

In the case of microstrip odd modes or coplanar waveguide even modes the corresponding expressions are:

$$\tilde{E}_y^{TM}(k_n) = \int_0^a E_y^{TM}(x)\sin(k_n x)dx , \tag{4}$$

and

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$$\tilde{H}_y^{TE}(k_n) = \int_o^a H_y^{TE}(x)\cos(k_n x)dx \quad . \tag{5}$$

The solution of the multiple boundary layer problem yields the representation

$$\begin{bmatrix} j\tilde{E}_z \\ \tilde{E}_x \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} j\tilde{J}_x \\ -\tilde{J}_z \end{bmatrix} , \tag{6}$$

$$\begin{bmatrix} j\tilde{J}_x \\ -\tilde{J}_z \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} j\tilde{E}_z \\ \tilde{E}_x \end{bmatrix} \tag{7}$$

where each term in equations (6) and (7) is found as follows:

$$G_{11} = G_{22} = \frac{\pm k_o k_n \beta}{\gamma_n^2 \omega\varepsilon_o}\left[F_1(k_n,\beta) + F_2(k_n,\beta)\right] \tag{8}$$

$$G_{12} = \frac{k_o}{\gamma_n^2 \omega\varepsilon_o}\left[k_n^2 F_1(k_n,\beta) - \beta^2 F_2(k_n,\beta)\right] \tag{9}$$

$$G_{21} = \frac{k_o}{\gamma_n^2 \omega\varepsilon_o}\left[\beta^2 F_1(k_n,\beta) - k_n^2 F_2(k_n,\beta)\right] \tag{10}$$

$$\Delta = G_{11}G_{22} - G_{12}G_{21} \tag{11}$$

$$H_{11} = G_{22}/\Delta \quad H_{12} = -G_{12}/\Delta \quad H_{21} = -G_{21}/\Delta \tag{12}$$

$$F_1(k_n,\beta) = \left[1/f_{t1} + (\alpha_t^{(2)^2} f_{t2} f_{t3} + 1)/(f_{t2} + f_{t3})\right]^{-1} \tag{13}$$

$$F_2(k_n,\beta) = \left[1/g_{y1} + (g_{y2} g_{y3} + \alpha_y^{(2)^2})/ \left\{\alpha_y^{(2)^2}(g_{y2} + g_{y3})\right\}\right]^{-1} \tag{14}$$

$$\gamma_n^2 = \beta^2 + k_n^2 \tag{15}$$

$$\alpha_y^{(i)} = \sqrt{\left\{(\gamma_n/k_o)^2 - \varepsilon_y^{(i)}\right\}/(\varepsilon_y^{(i)}\varepsilon_t^{(i)})} \tag{16}$$

$$\alpha_t^{(i)} = \sqrt{(\gamma_n/k_o)^2 - \varepsilon_t^{(i)}} \tag{17}$$

$$f_{ti} = \tanh(k_o \alpha_t^{(i)} h_i)/\alpha_t^{(i)} \tag{18}$$

$$g_{yi} = \alpha_y^{(i)} \tanh(\varepsilon_t^{(i)} k_o \alpha_y^{(i)} h_i) \tag{19}$$

Now the current/slot field distribution is expanded in unknown basis functions. Basis functions are chosen so that the edge effect is properly included; thus

$$\begin{Bmatrix} -J_z(x) \\ E_x(x) \end{Bmatrix} = \sum_{k=1}^{N} C_k f_k(x) \ , \ \begin{Bmatrix} jJ_x(x) \\ jE_z(x) \end{Bmatrix} = \sum_{k=1}^{M} D_k g_k(x) \tag{20}$$

where $g_k(x)$ and $f_k(x)$ are only defined on the strip for microstrip line and on the slot for slot line/coplanar waveguide case. C_k and D_k are unknowns with

$$f_k(x) = \left(\frac{x-s/2-w/2}{w/2}\right)^{k-1} \text{ and } g_k(x) = \left(\sin \ k\pi \ \frac{x-s/2}{w}\right).$$

Galerkin's method yields (N + M) x(N + M) eigen-equations to be solved. Once β is determined for given k_o, then the effective dielectric constant is obtained as $(\beta/k_o)^2$. If one of the coefficients is fixed, the other unknown coefficients are also determined.

III. NUMERICAL RESULTS

The method has evolved into an algorithm which yields results for any of the transmission line systems typically used in practice. During the presentation a discussion will be given describing the analysis and the numerical techniques which have been adopted. A few examples will be addressed here as an indicator of the versatility of this approach.

a. Inverted Coupled Microstrip Case:

The geometry considered is that of Figure 2 where the effective dielectric constant and phase velocity are shown for the even-odd mode cases supported by the structure.

b. Coplanar Waveguide Case:

The situation of interest here is that of Figure 3. At the low frequency limit agreement is obtained with the quasistatic results of Kitazawa and Hayashi (5).

c. Slotline:

The dispersion characteristics of the slotline are shown in Figure 4, together with the particular structure considered for this example.

A thorough description of the algorithm as well as numerous applications to additional structures on anisotropic substrates will be discussed during the presentation.

IV. CONCLUSIONS

A method is described which has evolved into a generalized algorithm for the computation of the dispersion properties of single and coupled microstrip, inverted microstrip, stripline, slotline and finline, as well as coplanar waveguide circuits on anisotropic substrates. The method yields results with excellent accuracy.

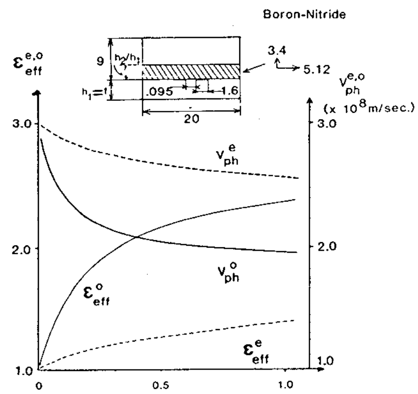

Fig. 2 Quasi-static h_2/h_1 Characteristics of Inverted Coupled Microstrip line on Boron-Nitride Substrate

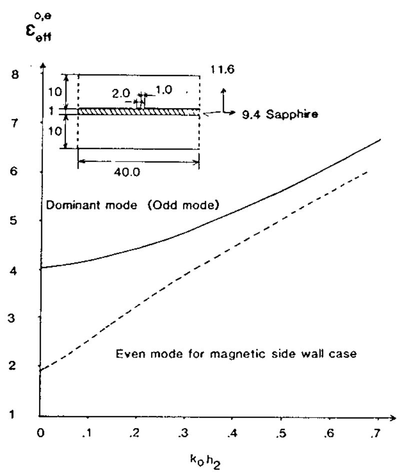

Fig. 3 Coplanar Waveguide Geometry and Dispersion Curve

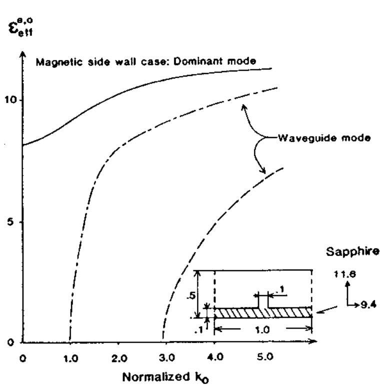

Fig. 4 Dispersion Characteristics of Single Slot and Waveguide modes

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